Book 3 Chapter7 – Mathematical and scientific thinking

Introduction

Development => increased understanding of language and physical environment occurs as organisation and systematisation of knowledge is carried out. Shown by:

-> Development of categories – both explicit knowledge of categories and the similarities/differences between aspects

-> Verbal labels are applied to categories and our own internal mental states

-> language is used to communicate ideas and desires

-> through language, children acquire knowledge about how other people think and see the world

-> this allows the creation of hypotheses about how language works, how the world works and how others think.

This process is the way we move towards creating representations of the world that are symbolic. Such symbolic representations allow us to deal with things that are imaginary, unknown and hypothetical. Such a skill is a pre-requisite for being able to think in mathematical and scientific terms.

Maths and science is not just about learning facts and procedures, but the understanding of a system which allows the generation of new facts that have not been taught explicitly. Children sometimes misunderstand the 'adult' system (e.g. large numbers), but still apply their own system to their generation.

The 'world is round' is understood as a fact early on, but it also has underlying principles not so well understood at first that explain that knowledge.

Nussbaum and Novack – bottles of liquid, dropped balls at poles and equator. 6 and 8 y.o. know that that world is round but are unable to extend this to the implication that liquid will not 'fall' out of bottles at the south pole – alternation between 'flat' and 'round' earth view.

To understand maths and science is to be able to think about a problem in an appropriate way, not rote learning of facts.

Development of Mathematical Understanding

Piaget's studies on mathematical reasoning appear to support the conclusion children find the acquisition of mathematical understanding difficult. Subsequent research exploring the factors that seem to mediate understanding of mathematical concepts comes to different conclusions.

Piaget's research

Sees children's learning as an attempt to view knowledge as structured and generative. E.g. understanding of numbers is not about learning their labels, but realising sets of objects were invariant unless new objects are added or removed.

4-5y.o. realise that adding increases #items in a box, but **Piaget and Szeminska** found spreading objects out meant children concluded there were more of them. Putting them closer together elicited responses that there were fewer of them.

Conservation – young children do not understanding that only the addition of more objects (coins) meant the quantity increased. The discovery of invariance happens around 6-7y.o. – so important it marked for **Piaget** a new stage – concrete operations.

Proportional reasoning is the next landmark – the recognition that the relationship between two variables remains the same even though both quantities change – formal operations stage.

Limitations of Piaget's ideas

Content, representation and social situation have all been shown to impact the success people have when solving equivalent problems. E.g.

Content – 5p per sweet => fixed ratio; yet the relationship of the length to the width of a rectangle (also a fixed ratio) is not so obvious to most.

Representation – **Nunes** found 12-13 y.o. solve -ve number problems more easily orally than when written down.

Social situation – **Lave** – adults more successful at problems in a supermarket than on a written test.

Vergnaud reconciles this paradox through his 'theory of conceptual fields'.

To successfully analyse a mathematical concept, its invariant properties, the situations that give the problem meaning and the symbols used to represent it need to be considered.

Understanding addition

Children's understanding of addition/subtraction is more than 9-4=5 – knowledge of number facts, but also relies of their ability to analyse a situation (**Brown, Vergnaud**).

Simplest types of these problems are elements added to or removed from sets – transformations.

Easiest to do are join or separation of two sets – 3 boys + 2 girls = 5 children.

Comparisons are harder – Mary has 5 books, Tom 3, how many more books does Mary have?

Therefore explains why 5+4 is easier than ?+4=9 – inverse problems are more difficult than direct problems.

Bryant et al - demonstrated that an understanding of inverse problems is not all or nothing; inversion with identity (e.g. removing blocks in front of a participant) is easier than doing it without the identity present.

Counting Strategies and Understanding

Groen and Resnick - the ability to 'count on' from a number is not necessary for children to improve efficiency. 5 pre-school children, 4;10, taught how to add using blocks and counting from 1. When asked to solve a large number of problems, they 'invented' both counting on and 'counting from larger' - demonstrating an understanding that addition is commutative.

Cultural practices - Oral vs Written

Carraher et al - work with Brazillian children, 5 participants. Problems solved orally as street vendors 98% success; in the classroom 74% correct on narrative problems; 37% correct on formal computations.

Implies symbolic systems are not accessories to reasoning but that they mediate it - i.e. they change the nature of the activity.

Nunes et al in a later study found similar results and that written arithmetic led to larger errors. Qualitative data also supports - Ev - could not do 100/4 on paper, but able to easily do it in her head as a divided by 2 problem twice.

Conclusion is that oral arithmetic retains the 'human sense' of the problem, therefore success is greater.

Different types of number

Whole numbers easier to grasp than fractions for both children and adults, as a major conceptual shift is needed for comprehension (**Behr; Kieren; Inhelder; Piaget; Gelman & Meck …**)

Integers are part of everyday life, fractions tend not to be.

Piaget - cut cake in half diagonally or vertically - is each piece of cake the same size? Children rely on experience rather than the logic of dividing the whole into two for their answer.

Nunes confirmed in a written task - 45% of 8y.o. and 20% 9y.o. fail.

Mack - 12y.o. says 1/6 smaller than 1/8 as '6 is smaller than 8'. However, no difficulty in recognising a pizza cut into 8 pieces has smaller slices than one cut into 6.

Nunes and Bryant confirms; sample of 142 aged 8-10 given two comparable maths problems.

2 boys and 3 girls each share an identical pie; 89-96% correctly identify boys get more pie.

Compare ½ to 1/3; success rate drops to 11%, 31% and 59& for 8,9 & 10 y.o.

Development of scientific reasoning

Logical aspects, situations and representation of concepts affect success and understanding, as with maths.

Understanding Physics

McClosky et al - Physics students more successful at working out on paper when to drop a ball into a basket when running towards it but no more successful at doing the task in real life. Indicates physics students have qualitatively different thinking about forces etc. than non-students and that we do not need a real understanding of physical principles in order to operate as human beings.

Science in the classroom

Difficulties of making distinctions are apparent - e.g. temperature != heat, but this is not usually important in everyday life.

Piaget and Inhelder - pioneering work. Asked what happened to sugar when dissolved in water. Younger children believe 'it disappears'; 11-13y.o. aware that as the water now tastes sweet it must still be present in a different form. Older children therefore appear to invent an atomic theory of physical quantities.

Driver et al - had children produce illustrations of what happens to water as solid, liquid and gas - shows the difficulty of moving from a world of observables to a conceptual world of the unobservable.

Chi et al - not all scientific ideas are based on matter and properties; many are about processes. Examples are light, heat, electric currents.

Reiner et al - children often treat heat and temperature as synonymous - they are substances with the property 'hotness' and are unable to think of heat as existing independently from objects.

Nunes et al - many scientific concepts are intensive rather than extensive qualities (extensive = measurable through a single unit).

Taste - intensive. Studies on dropping a sugar cube into different quantities of lemon juice show 7 and 9 y.o. respond no better - difference in correct conclusions as to taste no better than chance. If lemon juice quantity remains constant and sugar added varied, correct responses are close to 100%.

Scientific principles - parsimony and consistency

Parsimony - smallest number of assumptions to explain the largest number of phenomena. Not generally used in everyday life but key to scientific thinking.

Vygotsky - scientific concepts learnt by going from the general to particular; but in everyday life people generalise from individual observations.

Consistency - not always used to validate everyday knowledge - e.g. **Hatano** found young children can give answers to problems involving heat but inconsistently.

e.g. adding cold water to a hot bath cools it; but say 1 litre at 20C added to 1 litre at 60C means the water will be 80C (not 40C). **diSessa** used different examples to make the same conclusion; **Posner** differs as he argues their theories are consistent, if different and less powerful than adult theories.

Cognitive development and the acquisition of scientific and mathematical principles

Formal operations & scientific understanding

Inhelder and Piaget – pioneers, scientific understanding develops only as part of formal operations stage. Two questions arise:

(i) Is it necessary for adolescents to understand scientific reasoning to do maths/science at school?
(ii) Once they can reason scientifically, does understanding of scientific concepts improve?

Inhelder and Piaget – development of appropriate cognitive structures required before concepts like density can be mastered. Claim that children can discover many scientific principles alone as their power of reasoning will enable generalisation to other contexts.

Support for cognitive structures being important from **Piburn**. Performance in tests of if-then (proportional) reasoning performance correlates with science grades.

Cheng and Holyoak found people think about if-then propositions in relation to both content and context i.e. different inferences are made if the 'then' is a prohibition than if it is a promise.

Conceptual changes in individuals

Instruction based on telling children how scientists think is not very successful; definitions and formulae are learnt but their original pre-conceptions remain.

Piaget argued cognitive change should be expected from the results of children's own intellectual activity. Changes in thinking results in conflict; the process of *equilibration* is required to cope with the disequilibrium induced. Believed that children needed to be active in their own problem solving to challenge their current reasoning. However, sometimes this doesn't result in a change – e.g. reclassification of a carton of milk as 'light' from 'heavy' because it floats may happen.

Conceptual change as a social process

Piaget – theories translated into a teaching approach known as *discovery learning*. Lone learner route originally. Subsequent research showed interaction with peers may also be important – e.g. **Perret-Clermont** and **Doise & Mugny**.

Howe et al – support from 8-12 y.o. in conflict/non-conflict groups w.r.t. motion down a slope – conflict group had better understanding 4 weeks after the exercise. 12-15 y.o. – understanding the path of falling objects showed pairings with different original understandings engaged in more discussion, resulting in a better understanding afterwards.

Vygotsky, Bruner, Luria – conceptual change is therefore not solely from a child's own activity, but also as a result of their social interactions.

Vygotsky – ZPD concept. Knowledge is not assimilated (**Piaget**), but appropriated through the use of cultural tools.

Newman et al – discussion of a lesson on acids and bases. Children paired up with different chemicals to mix. Teacher has attempted to minimise errors by offloading the tasks not centred on the learning exercise by the design of the lesson. An illustration of how some elements of a system of knowledge may be used before a full appreciation is obtained.

Conclusion

Societies have developed systems of knowledge that through social participation children can use but do not necessarily understand in full.

Some problems remain – e.g. use of fractions is hard even for many adults.

Children organise their environment based on inferring rules from experience; but initial hypothesis formation is limited by the experiences they have had.

Tim Holyoake 2009, http://www.tenpencepiece.net/