

Introduction

Reasoning and logic

History: Aristotle - first system of logic. Boole - 'The laws of thought' - set of rules for drawing inferences from if ... then statements. **Inhelder and Piaget** - logical reasoning starts when we reach the formal operational stage (stage theory of cognitive development).

1st theme - does logic provide a good model of human reasoning? Even if it does, we may not use this strategy as perfect performance may require too many cognitive resources - therefore, we would expect to see errors of logic.

2nd theme - is logic appropriate to describe human reasoning at all? e.g. Boole devoted more than half of 'The laws of thought' to probability theory. If we use probability to reason, then perhaps our reasoning is **error free but probabilistic** - rather than **error prone but logical**.

3rd theme - experimental work shows we make many errors if we judge reasoning by the standards of logic. So if we are irrational, are we really responsible for our actions and where is the boundary between sanity and insanity?

Reasoning in everyday life

Human reasoning is ubiquitous - even in mundane tasks like inferring that if our partner's car is on our driveway when we arrive home, someone must have driven it there and so they will be in the house - meaning that to open the front door ringing the bell is a good strategy,

We use given information - **premises**, to infer new information - **conclusions**. However, simply believing a premise does not compel you to believe the conclusion. Relationships between premises and valid conclusions are central to logical/deductive reasoning.

Chapter 12 - Reasoning

Deductive reasoning and logic

Accounts for the relationship between the premises and conclusion of a deductive argument.

Logical connectives

Descriptive clauses - e.g. 'John has a cold'
Structure building words - e.g. and, or, not, if...then - known as connectives.

Conditionals - of the form if p then q
p = **antecedent**
q = **consequent**

When are arguments logically valid?

Determined with the help of a 'truth table':

| Possibility | Connectives | | | | | |
|-------------|-------------|---|-------|-------|-------------|-------------|
| | Sentences | | not | and | if ... then | |
| | p | q | not p | not q | p and q | if p then q |
| A | T | T | F | F | T | T |
| B | T | F | F | T | F | F |
| C | F | T | T | F | F | T |
| D | F | F | T | T | F | T |

If you believe the premises of a deductive argument, then you are compelled to believe the conclusion.

Two examples:-

(i) **modus ponens (MP)**: if p then q is true (conditional) and p is true (antecedent), then q must also be true (consequent).

(ii) **modus tollens (MT)**: if p then q is true (conditional) and not q is true, then not p must be true too.

Logically invalid inferences

Two examples:-

(i) **Affirming the consequent (AC)**: if p then q is true and q is true, then p is true.

(ii) **Denying the antecedent (DA)**: if p then q is true and not p is true, then not q is true.

While such sentences may not be logically valid, it is possible they appear to make some kind of sense:

An AC example: If John has a runny nose, then he has a cold. John has a cold. Therefore, John has a runny nose.

Form and meaning in logic

MP and MT both have logically valid forms, as they draw logically correct inferences according to the truth table. It is the form of the argument, rather than its meaning, which allows us to make a logically valid inference.

One theory of human reasoning - **mental logic** - argues we use formal inference rules like MP and MT - so drawing inferences relies only on the form, not the meaning of an argument.

Another theory - **mental models** - argues we consider things like truth tables - in other words, our inferences rely on the meaning of the connectives.

Psychological theories of reasoning

Three general theories considered: **mental logic**, **mental models** and the **probabilistic approach**.

Mental logic

Also referred to as **formal rule** theories. Several different versions of a basic account - e.g. **Braine and O'Brien**; **Rips** - similar to **Piaget's** view that adult

thought is the operation of formal logic. The argument is that we execute rules similar to MP/MT. Failure to reason correctly is explained by people not possessing all of the formal rules in the truth table. Unless you can use all of these rules it means that some inferences will be far more difficult to make than others.

Mental models

Johnson-Laird; Johnson-Laird and Byrne - individuals are in principle capable of logical reasoning (c.f. mental logic approach) **but** argues reasoning takes place using pictorial representations of what sentences **mean**, rather than applying formal logic rules.

Connectives exclude possibilities - e.g. if p then q is true, 3 **possibilities** (this account calls them **mental models**) A, C and D exist. This implies that to interpret it properly, a mental model for each possibility is required. The **limitations of working memory** mean that this may not be possible - so we may instead have a preferred initial representation. How these models are manipulated therefore explains an individual's reasoning performance.

The probabilistic approach

Oaksford and Chater - logic is not a good framework for understanding how we make inferences. For example, according to logic, a conditional is true whenever its antecedent is false (possibilities C and D) - e.g. a good reason to believe the statement "If John has a runny nose then he has a cold" is that you do not believe that "John has a runny nose". Clearly, not believing this is not a good reason for believing the conditional statement.

Instead, according to probability, believing John does have a runny nose makes it very possible he has a cold - in other words, **an assessment of the conditional probability** leads us to the inference. e.g. if the probability is 0.9, then it can be written as:

$$P(\text{John has a cold} \mid \text{John has a runny nose}) = 0.9$$

Belief in the conditional is therefore a matter of degree - and is not absolute, as implied by formal logic - confirmed by **Evans et al.**

Important - as the definition of rationality changes. The first two theories imply that logical reasoning = rationality and errors = performance limitations; this theory implies probability theory = rationality.

Two main tasks - Conditional inference and Wason's selection task are often used to assess reasoning ability.

Conditional inference

Conditional inference task (**Evans; Taplin**) provide participants with a conditional sentence - the **conditional premise** (if p then q), along with facts relating to the antecedent (p) or consequent (q) of the sentence - the **categorical premise**.

Tasks are usually carried out with abstract stimuli - e.g. letters and digits - to investigate reasoning in such a way that prior knowledge does not interfere with the process. (But what about ecological validity?)

Schroyens and Schaeken - summarised 65 abstract reasoning experiments (e.g. if there is an A then there is a 2 and there is an A) - found:

| Inference Type | Proportion endorsing |
|-----------------------|-----------------------------|
| MP | 97% |
| MT | 72% |
| AC | 63% |
| DA | 55% |

Pair wise comparisons show all differences are significant - **so results not consistent with pure logic** (as MP/MT should be endorsed equally and AC/DA not at all)

Mental logic

Proponents (e.g. **Rips**) explain this result by suggesting people have the MP rule, but not the MT rule as more complex reasoning is needed.

e.g. If John has a runny nose, then he has a cold
John does not have a cold
Therefore, John does not have a runny nose.

So, we assume MP to start and find it results in a contradiction - i.e. John has a cold (the MP conclusion) contradicts the facts stated. So, instead, we figure that John has a runny nose is false and John does not have a runny nose is true. Known as **reduction ad absurdum** (RAA).

Similar strategy could explain results for DA and AC - as conditionals in natural language are often ambiguous and can be **bi-conditional** - if and only if - which is true only when if p then q and if q then p are true.

Means people that think like this should endorse both DA and AC - but as AC is easier (equivalent to MP), that explains the difference in acceptance between AC and DA

Conclusion - performance on the task is rational - but
(i) Some logical inferences are easier than others
(ii) Some interpret the conditional as a bi-conditional.

Mental models

| Conditional (if p then q) | | Bi-conditional (iff p then q) | |
|---|----------------|---|----------------|
| Initial model | Fleshed out | Initial model | Fleshed out |
| [p] q | [p] q | [p] [q] | [p] [q] |
| <small>There may be other models not currently being considered</small> | not p q | <small>There may be other models not currently being considered</small> | not p not q |
| | not p not q | | |
| MP | MP, MT | MP, AC | MP, DA, AC, MT |

[] indicate a term that cannot be paired with any other

Bottom line of table shows inferences that can be drawn from each model

The explanation for the **Schroyens and Schaeken** data is:

- (i) Mental models only represent the true possibilities
- (ii) There is a distinction between the initial and fleshed out models.

=> Different subsets of participants adopt these four mental models, leading to different proportions of the population accepting each of the four possible inferences MP, MT, AC and DA.

Probabilistic approach

Inferences are drawn according to how probable someone thinks the conclusion is for the given premises - **Oaksford and Chater**.

MP straightforward - $P(\text{cold} | \text{runny nose})$

If this is 0.95, then highly likely that people will endorse the consequent given the antecedent is very likely.

For AC, can use a form of Bayes' Theorem to calculate:

$$P(\text{runny nose} | \text{cold}) = \frac{P(\text{cold} | \text{runny nose})P(\text{runny nose})}{P(\text{cold} | \text{runny nose})P(\text{runny nose}) + P(\text{cold} | \text{not runny nose})P(\text{not runny nose})}$$

Probabilities can be calculated for DA and MT in a similar way.

Probabilistic approach gives a good fit to the **Schroyens and Schaeken** data:

| Inference | Probabilistic prediction | S&S Data |
|-----------|--------------------------|----------|
| MP | 0.88 | 0.97 |
| MT | 0.77 | 0.72 |
| AC | 0.68 | 0.63 |
| DA | 0.51 | 0.55 |

(Oaksford and Chater)

Everyday reasoning and the suppression effect

Everyday inferences are **defeasible** - e.g. if you draw the inference John has a cold because he has a runny nose and you then find he has hayfever, your inference is wrong.

Implies many everyday inferences are non-deductive - i.e. the truth of the conclusion is not determined by the truth of the premise.

All four inference types (MT, MP, AC and DA) can be defeated (suppressed).

e.g. for MP, a car having an empty petrol tank will defeat the inference that if the key is turned the car will start. Such exceptions are termed 'additional antecedents' - **Byrne**.

For DA and AC, 'alternative antecedents' can defeat inferences.

e.g. for DA, if told *if the key is turned then car starts and the key is turned* you could be tempted to endorse the DA inference: *the car does not start*. However, if

you are also given: *the car was hot-wired* you may be less likely to endorse the DA inference - and also less likely to endorse AC too (if you knew the car started you may not infer the key was turned as it could have started because it had been hot-wired).

Byrne demonstrated this by using explicit rules with additional information (additional and alternative antecedents) and found that:

1. Alternative antecedents suppress DA and AC but not MP or MT.
2. Additional antecedents suppress MP and MT but not DA and AC.

Cummins et al - able to demonstrate similar effects without explicitly presenting additional or alternative antecedents, implying people use memories to affect their reasoning performance.

Also demonstrated effects were **graded** - i.e. more additional/alternative antecedents result in greater suppression effects. This is a problematic result for theories that regard such effects as all or nothing (e.g. mental logic).

Experiments where participants were asked to rate on a scale of 1-7 how likely a conclusion was, such theories would predict individual participants providing all 1's or all 7's - but 50% of participants values are intermediate - again, not consistent with 'all or nothing'.

Mental logic

Byrne experiments - two conditional rules with the same consequents used. For additional antecedents, both conditional rules need to be true (**and**); for alternative antecedents, either rule needs to be true (**or**). People may therefore represent such premises as single rules with complex antecedents. MP and MT are not drawn for additional antecedents; DA and AC not drawn for alternative antecedents.

So mental logic does offer explanations for suppression effects but says they should be all or nothing - contrary to the **Cummins** data. **Rips** responds that mental logic provides a good explanation of deductive inference but not non-deductive inferences.

Mental models

The availability of counter-examples explains suppression effects in the mental models theory. As with mental logic, Byrne argues ‘and’ is used to represent additional antecedents and ‘or’ alternative antecedents. The following representations are therefore used:

| Additional antecedents | | | Alternative antecedents | | |
|---|-----------|-------------|--|-----------|--------|
| If the key is turned and there is fuel then the car starts | | | If the key is turned or it is hot-wired then the car starts | | |
| Turn | Fuel | Starts | Turn | Hot-wired | Starts |
| ... | ... | ... | ... | ... | ... |
| Turn | not(Fuel) | not(Starts) | not(Turn) | Hot-wired | Starts |

... means “there may be other models not currently being considered”

Fully fleshed out model for “and” will include the case where the key is turned but the car does not start as there is no fuel; for “or” it will include the case where the key is not turned but the car starts as it was hot-wired. These models therefore show why counter-examples need to be available.

Example invokes our prior knowledge of what affects a car starting - called the **principle of pragmatic modulation** by **Johnson-Laird and Byrne** - our LTM can modulate the interpretation of conditionals, so making counter-examples that much easier to find.

Mental models also suggest suppression should be all or nothing - but supplementing them with a validating search procedure as suggested by **Schroyens and Schaeken** and others, allowing LTM to be used, may either influence reasoning in an all or nothing or in a

graded, probabilistic manner - but such explanations imply people cannot be thought of as performing purely logical inferences - as if a conditional is ‘true’, then there is no need to search for a counter example!

Probabilistic approach

The presence of additional and alternative antecedents will have an impact on the conditional probabilities - **Oaksford and Chater**.

Information about additional antecedents - e.g. the petrol tank is empty - will have the effect of reducing the probability that the car will start and so suppress MT and MP inferences.

Alternative antecedents emphasise other factors - e.g. you can start a car without turning the key if it is hot-wired.

So, given that, $P(\text{car starts} | \text{key not turned})$ is higher than you first thought - which in turn reduces $P(\text{car does not start} | \text{key not turned})$ (probabilities always add up to 1). The probability of the car not starting given that you do not turn the key is what needed to be worked out to determine if you want to draw the DA inference - hence the observed suppression effect.

Wason’s selection task

Probably the most used task to assess reasoning from a psychological perspective.

The abstract selection task

Wason - used to understand how people assess if evidence is relevant to the truth or falsity of a conditional rule. Four cards used:

- A (p card) K (not p card)
- 2 (q card) 7 (not q card)

Each card has a letter on one side, a number on the other.

Participants have to select cards they would turn over to test if a rule is true or false - e.g.

If there is an A (p) on one side then there is a 2 (q) on the other side.

Have already seen that if p then q is only false when p is true and q is false.

So, only a card with A on one side but without a 2 on the other side makes the rule false.

Only the A card could have a number other than 2 on its reverse and only the 7 card can have an A on its reverse.

Logic therefore says that only cards A and 7 can falsify the rule (but not K or 2) - so these are the ones that should be selected.

However, experimentally, this pattern of results occurs:

| Card(s) selected | proportion of participants |
|-------------------------|----------------------------|
| A(p) and 2(q) | 46 |
| A(p) only | 33 |
| A(p), 2(q) and 7(not q) | 7 |
| A(p) and 7(not q) | 4 |
| Other combinations | 10 |

The choice of the q card made by more than half the participants is an example of **confirmation bias**, but is an irrational choice!

Evans and Lynch - work on the **matching effect** - shows apparent irrationality as well. Using rules containing negations, e.g. if A on one side then **not 2** on the other, A and 2 are still selected - but this is the correct logical choice. If they had been still showing confirmation bias, then A and 7 should have been selected. Conclusion is participants do not engage in this task rationally, but rather select cards matching the letters and numbers given in the rule!

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| <p><u>Mental logic</u></p> <p>Same approach taken as for the conditional inference task. Relies on:</p> <p>MP = p card (A) DA = not p card (K) AC = q card (2) MT = not q card (7)</p> <p>Given if A then 2, deciding to turn the A card (p) is the same as drawing an MP inference that there is a 2 on the other side.</p> <p>If participants mistake this for a bi-conditional - i.e. if 2 then A, turning the not A card is drawing the MT inference to predict not 2 on the other side (logically equivalent to DA on the original rule.)</p> <p>Problem - account predicts the proportion of people selecting the cards in this task should be the same as for the conditional inference - but this is not the case.</p> <p>A reason for this might be presentational - unlike the conditional inference task, there is no explicit conclusion given - participants are told if A then 2, shown a card with 7 on one side (so have to infer 7 is an instance of the category not 2) vs. if A then 2, not 2, therefore not A and being asked to say if the conclusion is appropriate.</p> <p>Evans et al - re-ran the conditional inference task with implicit negations - pattern of results for if A then 2 becomes close to the pattern of card selections in the selection task.</p> <p>Can also account for the matching effect - e.g. if not 2 is shown on the cards rather than 7, the matching effect is not detected - Evans et al.</p> <p><u>Mental models</u></p> <p>Also explains the results in a similar way to conditional inference task. However, each possibility is considered</p> | <p>to see if something on the reverse of the card can affect the truth/falsity of the rule. The frequencies of card selections are dependent on the proportions of participants adopting the different interpretations.</p> <p>Matching effect given the same explanation as in mental logic - i.e. it is a product of having to process implicit negations - Evans and Handley; Johnson-Laird and Byrne)</p> <p><u>Probabilistic approach</u></p> <p>Suggests selecting the p(A) and q(2) cards is not irrational, but the optimal response - Oaksford and Chater.</p> <p>Central argument is people look for the most informative evidence when trying to decide if a rule is true. If there is a relationship between p and q they are dependent - if not, they are independent.</p> <p>These two possibilities are given an even chance (0.5) of being true. Saying which cards to turn results in an (expected) information gain - reducing uncertainty as to which hypothesis is true.</p> <p>This model therefore explains the findings as a rational consequence of attempts to identify the most informative data.</p> <p>The deontic selection task</p> <p>Deontic reasoning - concerned with what you should/should not do. Very controversial research findings - as <u>evolutionary psychology argues it reveals the effects of a range on innate cognitive modules</u>.</p> <p>Work on the selection task appeared to <u>sometimes</u> show logic-like performance if real world examples, as opposed to abstract examples were used - e.g. the p and not q cards are selected for rules like:</p> <p>If Johnny travels to Manchester, then <u>he takes</u> the train</p> | <p>and examples like:</p> <p>if you are entering the country, then <u>you must have</u> a cholera inoculation</p> <p><u>reliably</u> produced logic-like performance (Cheng and Holyoak)</p> <p>Difference is accounted for by the use of indicative conditionals (<u>descriptive</u> of the world and how people behave) vs deontic conditionals (<u>prescriptive</u> - state how people should/should not behave).</p> <p>Deontic conditionals remain in force regardless of the number of violators - a 'must have' rule remains 'true' regardless of the number of law breakers - it is an obligation rule. So logical truth has little to do with how people evaluate them - even though people 'correctly' select the p and not q cards.</p> <p>Cosmides - deontic rules can also be framed as permission rules - (allowed, but not obliged) - e.g.</p> <p>if you have a cholera inoculation, they <u>you may enter</u> the country</p> <p>If participants are looking for potential violators, then they should pick the not p and q cards - and this is what was observed by Cosmides, and replicated by others (e.g. Manktelow and Over). <u>This result is not predicted from logic</u>.</p> <p><u>Mental logic</u></p> <p>Practitioners have not explicitly address deontic selection, but in principle, it is possible to extend it to account for deontic inferences (Manktelow and Over).</p> <p><u>Mental models</u></p> <p>Explanation offered by this account is continuous with the explanation of Wason's selection task. People represent possibilities and identify counter-examples. The distinction in the deontic selection task is what is</p> |
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deontically possible/permissible does not correspond to what is factually possible - **Johnson-Laird and Byrne**.

For the statement:

If you are entering the country, then you must have a cholera inoculation

all four truth table cases are factually possible. However, the deontic rule says it should not happen. The argument is that we tag different possibilities to indicate if they are or are not permissible.

Probabilistic approach

Adopts a decision-theoretic framework as proposed by **Manktelow and Over**. Decision theory argues people make choices to maximise expected utility.

This task asks people to place a high value on instances of unfairness - i.e. when people try to enter the country without an inoculation.

So as 'entering' (p) and 'no inoculation' (not q) has by far the highest utility, you should expect people to pick the p and not q cards.

The same predictions about people's behaviour on this task are made by this approach as for the mental models approach, but will also be sensitive to manipulations of utility and probability - **Kirby; Manktelow et al** - provide some evidence to support this view.

Evolutionary psychology

=> cognitive mechanisms are innately specified and have adapted under the evolutionary pressure encountered by early humans.

Deontic reasoning might be controlled by an innately specified mechanism - **Cosmides; Fiddick et al**.

Argue that the effects observed can be explained by assuming a cognitive module for social contracts which governs social exchanges - as an obligation rule or permission rule - e.g.

If you accept the benefit you must satisfy the reqm't

If you satisfy the reqm't then you can have the benefit

So this module would not be sensitive to logic, as what is important for survival is not what is logically true, but whether you get cheated.

Such cognitive modules are invoked automatically and override domain general reasoning processes. They are also domain specific and so will only apply in situations where there is a clear benefit-requirement relationship.

Evidence - **Cosmides**

If a student is to be assigned to Grover High School, then that student must live in Grover.

One set of participants told going to Grover High (p) was benefit compared to going to Hanover High (not p); another set not told this.

Far more selected p and not q when the benefit was explicitly mentioned (p = benefit; q = requirement) - showing that the obligation form of a rule is not by itself sufficient to trigger this response.

Fiddick et al - automatic understanding of social exchange situations exists even in the absence of explicit rules.

First condition - 'if you give me potatoes, I will give you corn'. Second condition - told to imagine they were farmers who was told by a neighbour 'I want some potatoes' to which they reply 'I want some corn'

Participants provided with four cards corresponding to four people marked:

(i) you gave this person potatoes
(ii) you gave this person nothing
(iii) this person gave you corn
(iv) this person gave you nothing

Participants asked if any of the people represented by the cards had cheated them. Both groups perform equally well - either because people interpret the rule-less version as having the rule, or, as **Fiddick et al** argue, the rule-less scenario can only be translated into one rule consistent with a social exchange which produces the observed results.

Deontic selection => requires appropriate specification of benefits and requirements; is independent of the use of a conditional rule.

Conclusion

All of the principal theories of reasoning do fairly well at explaining experimental results, despite the differences in their approaches.

Competitive (next page) and integrative evaluations are possible.

Integration, dual processes & individual differences

Two types of rationality? - rational in one sense when reasoning conforms to logical standards; rational in another when reasoning helps to achieve a real life goal - **Evans and Over**.

Stanovich and West - individual differences explanation - ability to make logically correct responses on the selection task associated with high IQ. High IQ individuals can interpret the task logically; for remaining participants IQ correlates with non-logical but standard p and q card response - **Newstead et al**. Suggests a dual-process theory - maybe unconscious reasoning mechanisms that operate in accordance with probabilistic standards of reasoning, which higher IQ individuals can consciously override.

| Mental logic | Mental Models | Probabilistic | Evolutionary Psych. |
|---|--|---|--|
| Advantages | | | |
| Formally well specified, mathematical proof for its predictions are available | Range of coverage is broad for the available data (c.f. depth for mental logic) | More of our reasoning can be cast as rational. But have to take into account our prior k'ledge – e.g. rarity assumption | |
| Rational reasoning is a fully logical conception | More researchers focus here than in any other area | Predictions on manipulating probs and utils affect reasoning are usually confirmed by data. | |
| Disadvantages | | | |
| Narrow coverage – not easy to see how the approach can be applied to observations such as graded data in suppression experiments; has not been applied to explain <u>deontic</u> reasoning | Pragmatic modulation not consistent with graded effects – if a conditional is believed true, then surely there is no need to look for counter examples | Coverage small c.f. mental models. Only provides an account of how cognitive system behaves given specified inputs – nothing to say on representations / processes involved. | |
| Other issues and notes | | | |
| <p>Rips – has proposed a distinction between deductive and non-deductive reasoning. Argues people evaluate arguments based on their inductive strength and deductive validity.</p> <p>Integration may therefore be possible between the theories – mental logic for clear-cut cases of deductive reasoning, other theories with the remainder.</p> | <p>The 'crucial experiment' which falsifies mental logic while supporting mental models has not been found.</p> <p>Difficult to understand what is says about rationality – extends beyond the scope of standard logic – but in such instances don't show any approximation to logical/mathematical theory.</p> <p>Evidence from cognitive neuroscience – Goel et al – conditional inference in left hemisphere language, rather than right hemisphere imagery systems. So implies mental logic approach instead.</p> | <p>This theory suggests that our standard of rationality is faulty, i.e. reasoning should be judged by probabilistic standards rather than logical ones.</p> | <p>Suggests we have two innate cognitive modules – one for social contracts; the other for reasoning about hazards.</p> <p>Stone et al – <u>neuro-psychological</u> evidence for brain damage causing impairment in social contracts but unimpaired hazard management. However, until a double dissociation is found, evidence remains inconclusive that such a distinction exists.</p> |