Chapter 10 - Problem Solving

Introduction

Problem solving is: essential, familiar, pervasive. Occurs naturally - e.g. tool use, foraging; structures life in meaningful ways. Important to understand how we solve problems so that (safety) critical errors in tasks can be designed out.

Problem solving research incorporates material from other cognitive psychology disciplines - e.g. linguistic skills, visual perception, memory and attention.

The concept of 'representation' is central to cognitive psychology [chapter 17] - the assumption is that internal representations of information are used during problem solving.

Problem solving occurs over a period of time, so the processes and representations involved interact with others - often involving processes of reasoning, judgement and decision making.

It is often the case that initial attempts to solve a problem fail, so we have to turn to other sources of help (e.g. manuals, experts).

Some problems are unwelcome - e.g. dealing with an oil spill; others are for pleasure - e.g. sudoku.

Much of the research in this area addresses the question of understanding which factors influence the construction of a problem representation.

What is a 'problem'?

Duncker - "***a problem exists when a living organism has a goal but does not know how this goal is to be reached***"

e.g. Finding a babysitter - several ways to do this; some more novel than others; availability of solutions may vary depending on the context (how much time is there to find a solution), social setting and culture.

Finding the best move in a board game is more to do with knowledge, skill and experience - but the player's motivation is part of it to (does it matter if I lose?)

Finding the best layout for a paper pattern on material involves perception - seeing how to lay the pieces out in the optimal way - and some solutions may be better than others (quicker to cut out; less material used; pattern matches at the seams …)

Therefore, different problems are affected by different factors - **internal** (e.g. motivation) and **external** (e.g. culture)

Protocol analysis in problem-solving research

Its legitimacy depends on the assumption that information represented in working memory can be verbalised directly (if held in verbal form) or through transformation (if it is non-verbal). Long term (memory) information needs to be moved to working memory before it can be reported on.

The protocol (verbal account) resulting from thinking aloud is assumed to preserve the order in which information has been handled.

Other assumptions are that cognition=information processing; information is stored in different areas of memory; recently acquired information is held in working memory.

e.g. use - add 63 and 37 together and think aloud while you're doing it. Demonstrates different people use different methods to get to the results and can also provide information about the strategies we use to solve problems.

Key point is that mental processes can be inferred through the analysis of verbal reports - not that thinking aloud is a direct externalisation of cognition.

Protocol analysis in medical diagnosis

Debate centres over the extent to which expert clinicians use biomedical knowledge to diagnose.

Lesgold et al - extensive use made.
Boshuizen and Schmidt - very little use made.

Gilhooly et al hypothesis: When experts can use contextual information (age, lifestyle etc.) then use of biomedical knowledge is suppressed.

Experiment - asked clinicians of varying expertise (registrars, house officers, students) to interpret ECG traces.

Results: The more experienced, the more accurate the diagnoses. Protocol analysis showed more use of biomedical knowledge was made by expert clinicians. Also that more use of clinical knowledge was made.

Conclusions: 1 - This research resolves differences in the literature - increased use of biomedical knowledge is correlated to expertise if a shortcut cannot be use to aid diagnosis.

2. Protocol analysis is a useful tool in real-life problem solving scenarios.

'Simple' problem solving

Looks at problems that require no extensive background knowledge to solve them - also know as puzzles. Key issues for this type of problem solving are representations, the way problems can be manipulated to affect representation and problem solving performance.

The Gestalt legacy

Hallmark - the **insight** phenomenon - "aha!" moments. **Gauss** (cited by **Hall**) - fast solution to adding up 1..100 by recognising a pattern - i.e. 50x101 = 5050.

Duncker – the process of restructuring a problem investigated by the X-ray problem – sub-goal of lowering the intensity of an x-ray and using it from several different locations killed the tumour at its focus without damaging the surrounding tissue – the goal.

Insight not achieved if participants are trapped by misleading representations that present solution – the **'set' effect**.

e.g. Solving multiple problems of arranging water in jars of different capacities where the solution is B-A-2C leads to difficulties in then solving a (simpler) problem with the solution A-C.

The *nine-dot problem* is similar – the layout of the task produces the difficulty.

Functional fixity occurs when an object has to be used in a new way to solve a problem. **Duncker** – mount candles side by side on a door so they can burn safely.

Two groups – one with tacks, matches, 3 candles and 3 small boxes; second group with same items but stored inside the boxes. Solution rate was much higher for the first group than the second – a failure of not being able to perceive the use of the boxes when presented as containers.

Real world example – the steam engine – used for pumping water from mines for many years before the idea of using it to power a locomotive was thought of.

Representation in puzzle problem solving

Representational effect is well attested in problem solving research. **Simon and Hayes** – Tower of Hanoi problem ('state-space' diagrams show its complexity); and their 'monster problem' – isomorphic to the Tower of Hanoi.

Same underlying structure to a problem means that they are isomorphic. 'Move' version – different sized monsters transfer globes of different sizes to each other according to a set of rules; 'change' version (isomorphic) is different sized monsters holding globes which have to be changed in size.

'Change' problem is harder to solve than the 'move' problem. Different representations appear to be constructed of the problems; the representation for the 'move' problem has easier operations than the 'change' version.

Zhang and Norman – theory to account for representational effects focuses on 'internal' and '*external*' representations.

'internal' representations imply a processing and representational burden as they have to be encoded and maintained; 'external' representations are not stated explicitly but are implied by the problem.

Internal rules for Tower of Hanoi include: Only one disc may be transferred at once; an external rule may be created if the discs are replaced by cups of coffee (coffee would spill out if a smaller cup was placed in a larger cup) – the environment is therefore providing the constraint.

External representations appear to make problem solving easier; but they change the nature of the task.

The information processing approach: problem solving as search

Problem solving may result in us finding a sequence of actions, or may require the discovery of a single action from a large set of possibilities. The information processing approach therefore sees problem solving as a **search** process.

Can be **forwards** – from the starting state, or **backwards** from the goal using **problem-reduction** or **means-end** approaches.

Example – booking a trip from London to New York can be broken into the sub-goals of finding a hotel, booking a plane etc. Problem is solved backwards by working from the goal and then completing the sub-goals that are found to achieve the goal.

e.g Tower of London task – search processes required involve holding goals and intermediate results in limited capacity working memory [chapter 9].

Gilhooly et al found that on this task, thinking aloud indicates that the limitations of working memory affect search, so that just one action is selected from all available at any step. A small sequence is built up before returning to the start stage and trying again – general strategy used is means-end analysis.

Similar findings from studies of the hobbits and orcs task (**Thomas; Simon and Reed**) and water jars task (**Atwood and Polson**).

Means-end analysis – reduce differences between current state and the goal. Not always the best strategy – as to solve the hobbits and orc task you have to make a move that appears to take you further away from the goal (**Thomas**).

Preference for new states – to avoid looping – is seen. **Davies** – memory alone is not used to determine this – but inferences are used to see if a potential state might have been one already used.

Information processing approaches to insight

Gestalt approach – restructuring a problem is the basis of insight. However, this has been criticised as not explaining how restructuring a problem takes place.

Ohlsson – provides some suggestions. Possible actions – operators – are generated from long-term memory and cued by the representation of the problem. If the initial representation is misleading, it leads to an impasse – no further progress can be made.

Three ways can be used to change the representation to resolve the impasse:

1. Elaboration - adding information - e.g. the solver in the 'candle' problem (**Duncker**) has to realise the matchbox can hold the candle.

2. Re-encoding - changing the way the problem is phrased, rather than adding information - a man can have married 20 women if he is the one doing the marrying, rather than being married himself.

3. Constraint relaxation - making the goal less restrictive than initially assumed - e.g. 9 dots problem can be solved if you can go outside the square of the dots when drawing the lines. (**Chronicle et al** also point out another source of difficulty is the application of a heuristic search to cancel as many dots as possible at once - can only solve the problem by an extended look-ahead). Another example - **Knoblich et al** - matchstick algebra with roman numerals. Found it was harder to break constraints on changing arithmetic operators than changing number values by moving matchsticks around.

Analogical problem solving

Often new problems are similar to ones we've solved before - so we can be pointed at a solution by the use of an analogy.

Analogies in problem solving

Spellman and Holyoak - participants readily accept analogies - e.g. Saddam Hussein was like Hitler; 'Domino' theory of communism.

Analogies can be used to develop understanding in some problem domains - e.g. heart as a water pump; atomic structure as the solar system; human and computer based information processing in cognitive psychology.

Studies have often used Duncker's X-ray problem. e.g. **Gick and Holyoak** - told participants a story about a general attacking a castle with small groups of men on all sides. Rate of solution was low for groups not given the analogy; higher for those told the story; higher still for groups told to use the story as a hint.

Keane - the closer the base story is to the target problem the more likely transfer is to occur - e.g. if the analogy is a surgeon treating a brain tumour.

Anolli et al - remote analogies are ineffective without a hint.

Dunbar - analogical paradox - in real life analogies depend on deep structural similarities; in the lab, studies show participants tend to use superficial features and have difficulty with deeper similarities.

Blanchette and Dunbar - public spending cut analogies task - participants readily draw on deep analogies that are not politically based. Argued **generating** analogies required participants to use structural, not superficial features and that naturalistic analogy studies have participants that understand the topic in some depth. In the lab, material is not familiar and there is little pressure to encode the story in a deep way. This may explain **Anolli et al** - providing a hint encourages a deeper structural representation of the base story to be created.

How do analogies work?

Structure-mapping theory (**Gentner et al**). **Analogical mapping** process establishes a **structural alignment** between base and target. e.g. atom - nucleus = sun; electrons = planets. Aspects that don't match (satellites) are omitted. Higher order relationships apply - e.g. less massive objects orbit more massive ones.

Gentner and Gentner - different analogies common for understanding electrical flow - fluid through pipes/flows of crowds through passageways. Electrical resistance mapped appropriately in both - e.g. pipe width / gates in passages. Those who used fluid analogies did better on battery problems; crowd analogies make it easier to solve resistor problems.

'Complex' problem solving

Definition: problem solving requiring extensive domain knowledge

Types:

adversary - e.g. chess
non-adversary - e.g. debugging programs, medical diagnosis, code breaking

Early chess studies

De Groot - 5GMs and 5 skilled players - thinking aloud; searching (ahead) to find a move. Hypothesis was GMs should look further ahead and conduct broader searches to find their move.

Results surprising - no reliable quantitative differences, but GMs pick better moves.

De Groot - **recall-reconstruction**: GMs replaced positions seen for 2-15s almost without error (91%); poorer players averaged 41%.

Chase and Simon - reconstruct while the original still in view. Stronger players replace more pieces per glance and appeared to use the relationship between pieces to help.

Experts therefore not only have more knowledge, but their knowledge is organised in meaningful and accessible ways.

Experts work forwards

Larkin et al - experts/novices solving physics problems. Protocol analysis showed experts **work forwards** (use information in the statement of the problem to work towards the solution). Novices use means-ends analysis – **working backwards** – starting at the goal to solve one part of the problem, then re-trace their steps, working forwards until they have a solution.

Expert/novice problem solving strategies appear to differ as experts use domain knowledge to generate a good problem representation – supporting the use of a working forwards strategy.

Experts have better problem representations

Chi et al – problem representation and categorisation. Experts/novices do not vary on quantitative measures of how many categories they produced for organising the problems or how long it took them to categorise.

Differences were qualitative – expert physicists use 'deep structure' to group problem types together – i.e. the underlying problem solving principles required; novices used 'surface structure' details – e.g. whether problems involved pulleys or levers.

Experts therefore seem aware of the common factors between problems in terms of how a problem can be solved.

Schoenfeld and Herrmann – similar study with mathematical problem categorisation – professors vs novices. Supports **Chi et al** – professors sort in terms of similarities in solution methods.

Chi et al – experts perceive solutions in around 45s – categories may therefore correspond to problem schemata - packets of knowledge - that can be used to solve a particular type of problem.

Experts become expert through extensive practice

Performance improves with practice in systematic and predictable ways – e.g. 'power law of practice', in **Snoddy** – mirror tracing of visual mazes research.

Three main classes of explanation as to why practice helps:

- Individual task components executed more efficiently
- Sequences of tasks executed more efficiently

… both these argue retrieval of declarative knowledge becomes more efficient – bigger units or chunks can be retrieved.

- Qualitative changes in representation of task structure occur

… the argument that the nature of the task changes as we improve – e.g. a shift from algorithm to memory based processing.

Ericsson et al argue 10 years required to attain high performance in chess, maths, violin playing, and similarly for sports, arts and sciences.

Simon and Chase – 3,000 hours to become a chess expert; 30,000 to become a master.

However – **Ericsson and Polson** – practice alone is not a guarantee of superior performance. Study of waiters – those who are able to remember orders best use more effective encoding strategies and as such are better than equally experienced peers who did not use the same strategies.

What a person does while they are practicing – as well as how much practice they do – is therefore important.

A modal model of expertise?

Chess studies – lots of research into the expert-novice paradigm. Model which emerged became known as the pattern recognition hypothesis.

Performance depending on a body of structured domain knowledge also applied to non-adversarial domains – e.g. programming (**McKeithen et al**), physics (**Chi et al**).

Results from such studies show a link between expertise and knowledge – a '**modal model**' – expertise depends on acquisition **and** organisation in LTM of domain-relevant knowledge/skill.

Sternberg – these observations about expertise are descriptive and lack explanatory power – over use of the paradigm simply suggests experts 'know more' than novices.

Prospects for problem solving research

Does expertise transfer?

Indications that the modal model was not complete came from further studies on chess skill.

Chess and memory skill

Holding – differently skilled players asked to memorise random chess positions. Players then asked to select best continuation moves.

Skill level unrelated to recall of random positions (as found earlier by **De Groot**), but best moves is correlated to skill level. Therefore, differences in memory alone do not account for good players finding better moves – instead, another factor is likely to be the ability to evaluate a position.

Holding – 50 players (class A to E) – set of test positions to evaluate for advantage and how strong it was. Better players more often right about game outcomes. Their evaluation of the next 'best' move correlated strongly with their skill vs the actual GM move.

Role of general and specific methods

Schraagen – asked participants to design expt around cola tasting - sensory psychology domain. Compared reasoning of domain experts vs design experts. As predicted by **Anderson**'s ACT* theory, found that when domain knowledge is lacking, skills of intermediate generality do transfer. In this case, domain experts generated better solutions but design experts reasoning was comparable to those of the domain experts.

Schunn and Anderson – studied expert scientists from different disciplines to examine if some skills were shared across domains. Protocol analysis and performance data shows that in the design of a memory experiment, domain experts design the best expts; domain and task experts differ in their **domain-specific** skills and task experts and undergraduates differ in **domain-general** skills.

Protocol analysis demonstrates a much larger set of **domain-general** skills are important to scientific reasoning.

Individual differences

Potential problem with studying expert-novice paradigm is that problems that are truly challenging for experts to solve could not even start to be solved by novices - therefore, experts tend not to be taxed by such studies.

Novices do not approach new problems with empty heads - they bring their own experience and strategies - so some novices will be better learners than others.

Are all learners the same?

A different approach is to study what novices can do - rather than look at what they do not have or cannot do.

People differ in their rate of learning => poor and good learners exist. So if novices don't have skill in an area before they start to learn, something else must mediate their speed of learning other than their skill in the area.

Thorndyke and Stasz – learning to map read study. Good learners encoded spatial information better; could accurately work out what they did and did not know; better able to focus attention on things they had not learned.

Green and Gilhooly – use of a stats package. Good learners made better use of worked examples and evaluated their learning. Poorer learners over-used worked examples, generated and tested more wrong hypotheses; ignored or failed to use error feedback.

Both studies suggest good learners make best use of **metacognitive** processes/strategies

Chi et al – various studies – on the role of explanation in learning. Analysed think-aloud from students studying science problems with equally proficient biology/physics knowledge.

- Good learners spontaneously self-explain more.
- Good learners use example to check their solutions; poorer ones use the examples to help them find the solution.
- Prompting people to self-explain leads to better problem solving

Renkl – self-explaining effect is not due to some students spending more time studying. Quality of self-explanations reliably predict success.

Self-explanations therefore seem to aid schema development - at the heart of all skill development.

Can we enhance the rate of skill acquisition?

Sweller et al – schema acquisition is slowed down by the use of means-ends analysis - paradoxical, as this is the strategy novices tend to rely on.

Hypothesised that over emphasis on a goal overloads a learner's system. Groups of students given a no-goal question (in 18s a car travels 305.1m; calculate as many variables as you can) switch to working forwards whereas those asked to specifically calculate its speed use means-ends strategies - goal biases the strategy.

Vollmeyer et al – found support for Sweller's claim that general problem-solving methods may help someone attain a specific goal, but also found that such methods do not promote the learning of the structure of a problem domain.

Non-specific goals aid learning as more hypothesis testing occurs - not (as **Sweller** claims) by the reduction of goal specificity in itself.

Green – reducing goal specificity alters the problem representation. The (changed) nature of the problem representation is crucial to performance - not the reduction in goal specificity. Also argues learning is different from problem solving - instructions that result in swift problem solving result in poor learning and vice versa.

Haider and Frensch – studies show the more skilled we are, the better able we are to ignore redundant information. However - not everyone behaves like this even after extended practice.

Green and Wright – extended this - if a choice of sources of information relevant to an answer is available, we tend to prefer the first encountered source. Information reduction then serves to reduce the processing of task-irrelevant, but also duplicated task-relevant information. Finding is at odds with theories like ACT*.

Do experts differ? **Draper** - experts not homogeneous - as all experts do not solve problems the same way. UNIX experts use different commands and sizes of command vocabularies. Little overlap between expert and novice UNIX command vocabularies. UNIX experts are specialist within a subset of commands. 1. Experts specialise in subsets of knowledge **Charness** - GMs do not know all opening, middle game and end game combinations. GMs specialise in particular subsets. 2. There are different kinds of expertise. e.g. physics experts use principles to solve problems; but there are no underlying principles to the UNIX o/s that are sufficient to know for UNIX experts to solve problems. However, both types of expert operate in domains where it is almost impossible to know everything there is to know - even though both are well-defined domains. Problem states become familiar over time; recognition is used to support problem solving. But, over-familiarity may lead to sub-optimal solutions being generated.	**Creative problem solving** - **Helie and Sun, 2010** Most theoretical models of problem solving concentrate on explicit processes; but in many ill-defined or complex problems a problem is found by sudden insight. Regular problem solving theories are unable to account for the failure / absence of deliberative strategy. **Helie and Sun** propose a general theory of problem solving - ***explicit-implicit interaction*** - EII. Based on **Wallas (1926)** four stage theory: **1. Preparation** - logic, reasoning, search - may result in a solution; if so, the process stops here. **2. Incubation** - can take a very long time - years - non-conscious 'work' on the problem happens. Incubating a problem has been shown empirically to increase the odds of finding an eventual solution. **3. Illumination/insight** - spontaneous manifestation of the answer, solution arrives in conscious thought. **4. Verification** - use of logic and reasoning to determine if the insight solution is correct - similar to preparation stage. Explicit - rule based - processing for stages 1 and 4; implicit - associative - processing for stages 2 and 3. Insight = transfer of the solution from implicit to explicit processing. **Helie and Sun** claim that they have been successfully modelled incubation/insight steps using the CLARION (connectionist) cognitive architecture and applied to human data.	